A Model of Financial Crises in Open Economies

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Abstract

The paper analyzes a small open economy with flexible exchange rates in which the financial sector is subject to confidence crises. A feedback between the health of the financial sector and the exchange rate amplifies the effects of crises. The presence of dollarized liabilities is crucial for this mechanism. Dollarized liabilities emerge endogenously in the model when domestic consumers expect a crisis with sufficiently high probability and switch from domestic to foreign currency deposits. In this framework, we analyze the role of a domestic lender of last resort. Precautionary reserve accumulation by the central bank facilitates effective lending of last resort, and can lead to a less dollarized financial sector and to a more stable exchange rate.

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1 Introduction

Banking panics are a common feature of financial crises, both in emerging and in developed economies. The main feature of a banking panic is that investors lose confidence in the short term liabilities of financial institutions, leading to a loss of funding for these institutions, to depressed asset values, and, eventually, to a contraction in credit and investment. In an open economy, with an open capital account, the problem is often compounded by a generalized loss in confidence in all domestic assets, leading to a capital flight. The central bank can respond to a banking panic by acting as a lender of last resort, providing emergency lending facilities to the banks in trouble. However, the combination of banking panic and capital flight makes lending of last resort particularly challenging in open economies.

Fixed exchange rate regimes offer stark examples of situations in which the domestic central bank lacks sufficient reserves to satisfy the demand of investors who are trying to convert domestic bank liabilities into foreign currency assets at a fixed rate. These situations eventually lead to a joint banking and currency crisis, i.e., a twin crisis. An open question is whether a regime of flexible exchange rates makes it easier or harder for the domestic central bank to act as a lender of last resort and mitigate a financial crisis. In this paper we explore this question by building a macroeconomic model with an intermediation sector that borrows both from foreign investors and from domestic consumers to finance purchases of risky assets.

The model features multiple equilibria which have the typical features of a banking panic with capital flight. In the bad equilibrium, all investors, domestic and foreign, are less willing to extend credit to banks and, at the same time, the economy overall experiences a current account reversal. We consider interventions by a central bank with limited resources, namely, foreign exchange reserves and a fixed amount of domestic fiscal revenue. We show that the presence of a fully flexible exchange rate and an inflation targeting regime does not eliminate the possibility of multiple equilibria.

Our analysis proceeds in two stages. First, we focus on the period in which the panic occurs, taking as given the economy’s initial conditions, including the assets and liabilities of the banks. We show that a flexible exchange rate regime is especially exposed to panics when three weaknesses are present: high leverage of domestic banks; high levels of foreign denominated liabilities; the fact that the fiscal resources that back up central bank interventions are in domestic currency. The ingredients behind these results are not new, leverage plays a similar role as in Gertler and Kiyotaki (2015) in a closed econ-
omy context, and the role of foreign liabilities is in line with the literature following the Asian crisis of 1997-1998, for example Aghion, Bacchetta, and Banerjee (2001), Corsetti, Pesenti, and Roubini (1999) and Chang and Velasco (2001). However, the connection between banks’ weak balance sheets and the real exchange rate is derived in a novel (and, we think, realistic) way and the role of the fiscal backing of the central bank is new.

In the second stage of the analysis, we take a step back and analyze the determinants of the banks’ balance sheets. Here, we focus on the endogenous choice of domestic consumers between domestic-denominated and dollar-denominated deposits. Our crucial result here is that this endogenous choice does not eliminate multiplicity, but actually adds a new layer to it. Consumers who anticipate banking panics, associated to large fluctuations of the exchange rate, tend to prefer dollar deposits, since they give them some protection against a devaluation. However, their preference for dollar deposits is exactly what pushes banks towards greater degrees of mismatch and thus to a higher probability of a panic. On the other hand, if consumers do not expect banking panics to occur, they have a natural preference for domestic deposits, because, due to the central bank’s inflation targeting, domestic deposits provide more stability in terms of domestic purchasing power. So the presence or absence of panics feeds back into the asset choices of consumers, because it changes the nature of exchange rate fluctuations. Without panics, exchange rate risk is just an unwanted additional source of risk for domestic agents. With panics, foreign assets become a good hedge, because the currency tends to depreciate exactly when the country is in a crisis. The endogenous nature of exchange rate risk in presence of financial crises is the main innovative contribution of our paper.


## 2 Model

The main ingredients of the model are as follows. We consider a small open economy that lasts three periods, \( t = 0, 1, 2 \), populated by two groups of domestic agents, households and bankers, who trade with a large number of foreign investors.

There are two goods: a tradable good and a non-tradable good. We assume that monetary policy keeps the domestic price level stable, so adjustments in the relative price
of tradables vs non-tradables lead to fluctuations in the nominal exchange rate. The
model features flexible prices, but movements in the nominal exchange rate matter be-
cause agents trade financial claims denominated both in domestic and in foreign currency.

The bankers act as intermediaries: they hold all the capital goods in the economy
and issue liabilities denominated in domestic and foreign currency. Therefore the price
of capital goods and the value of the banks’ liabilities affect bankers’ net worth and,
due to collateral constraints, bankers’ net worth affect real investment in the economy.
To allow for the endogenous determination of the price of capital goods, we have an
upward sloping supply of new capital coming from firms producing capital goods subject
to convex costs.

We now turn to a detailed description of the environment and to the definition of an
equilibrium. Along the way, we make a number of simplifying assumptions. Their role is
discussed in detail at the end of the section.

2.1 Agents and their decision problems

2.1.1 Households

Households enter period \( t \) with financial claims on domestic banks and foreigners. Let
\( a_t \) and \( a^*_t \) denote the households’ total claims, respectively, in domestic and foreign cur-
currency.\(^2\) The nominal exchange rate is \( s_t \), expressed as units of domestic currency per unit
of foreign currency. Except if otherwise noted, all prices are in domestic currency. House-
holds earn the wage \( w_t \) from supplying a unit of labor, inelastically, to the tradable sector.
They also receive every period an endowment of non-tradable goods, \( e^N \), and the profits
of the firms producing capital goods, \( \Pi_t \). Households use these resources to buy tradable
and non-tradable consumption, and to buy one-period claims in domestic and foreign
currency. Accordingly, their period \( t \) budget constraint is

\[
a_{t+1} + s_t q_t a^*_{t+1} + p_t^T C^T_t + p_t^N C^N_t \leq w_t + p_t^N e^N + \Pi_t + a_t + s_t a^*_t,
\]  

(1)

where \( q_t \) and \( q^*_t \) are the prices of one-period claims denominated in domestic and foreign
currency, \( C^T_t \) and \( C^N_t \) are consumption of tradable and non-tradable goods, and \( p_t^T \) and \( p_t^N \)
are their prices.

\(^2\)Both \( a_t \) and \( a^*_t \) are allowed to be negative, thus denoting a debtor position for the household.
The household flow utility function is $U(C_t)$, where $C_t$ is the consumption aggregator

$$C_t = (C_T^t)^\omega (C_N^t)^{1-\omega}.$$ 

Households choose state-contingent plans for assets and consumption levels in order to maximize expected lifetime utility

$$E \left[ \sum t \beta^t U(C_t) \right]$$

subject to the budget constraints (1) and the terminal conditions $a_3 = a_3^* = 0$.

We can simplify the households’ problem by separating the dynamic problem of choosing sequences for $C_t, a_{t+1}, a_{t+1}^*$ from the static problem of allocating consumption expenditure to tradables and non-tradables. Standard steps imply that consumption expenditure can be expressed as

$$p^T_t C^T_t + p^N_t C^N_t = p_t C_t,$$

where

$$p_t \equiv \omega - \omega \left( 1 - \omega \right)^{-\omega} \left( p^T_t \right)^\omega \left( p^N_t \right)^{1-\omega}$$

is the domestic CPI. Given the consumption level $C_t$, the optimal demands of tradables and non-tradables are:

$$C^T_t = \omega \frac{p_t C_t}{p^T_t}, \quad C^N_t = (1 - \omega) \frac{p_t C_t}{p^N_t}.$$

### 2.1.2 Bankers

Bankers run banks that hold the following assets and liabilities.

On the asset side, banks hold two types of capital goods, $k^T_t$ and $k^N_t$. The first, called $T$ capital, is used as an input in the production function

$$y^T_t = (k^T_t)^\alpha (l_t)^{1-\alpha},$$

and so it earns the rental rate

$$R^T_t = \alpha p^T_t (k^T_t)^{a-1},$$

since labor supply is 1 in equilibrium. The second, called $N$ capital, is used to produce non-tradable goods according to the linear production function $y^N_t = k^N_t$, so its rental rate is simply

$$R^N_t = p^N_t.$$
The two capital goods trade at prices $\Psi_T^t$ and $\Psi_N^t$. They do not depreciate in periods $t = 0$ and $t = 1$ and fully depreciate after production at $t = 2$.

On the liability side, banks issue one-period claims in domestic and foreign currency, denoted $b_t$ and $b^*_t$. Therefore, the banks’ net worth in domestic currency, is

$$n_t = (\Psi_T^t + R_T^t)k_T^t + (\Psi_N^t + R_N^t)k_N^t - b_t - s_tb^*_t. \quad (5)$$

and the banks’ budget constraint is

$$\Psi_T^t k_T^{t+1} + \Psi_N^t k_N^{t+1} = n_t + q_t b_{t+1} + s_t q^*_t b^*_{t+1}, \quad (6)$$
as banks use their net worth and newly issued claims to purchase the two capital goods.

There are two important sources of financial frictions in our model. First, only banks can hold capital goods. Second, banks face limits in their ability to raise outside finance. Namely, banks have to satisfy the following collateral constraint, which requires total liabilities to be bounded by a fraction of the $T$ capital held by the bank:

$$q_t b_{t+1} + s_t q^*_t b^*_{t+1} \leq \theta \Psi_T^t k_T^{t+1}, \quad (7)$$

where $\theta$ is a parameter in $[0, 1]$.

We assume that bankers only consume in $t = 2$, are risk neutral, and only consume tradable goods. Therefore, the bankers’ problem is to choose state-contingent plans for $\{k_T^{t+1}, k_N^{t+1}, b_{t+1}, b^*_{t+1}\}_{t=0,1,2}$ in order to maximize the expected value of $n_2/P_T^2$, subject to the law of motion for net worth (5), the budget constraint (6), the collateral constraint (7), and the terminal condition $b_3 = b^*_3 = 0$.

### 2.1.3 Capital goods production

The $N$ capital is in fixed supply, so in equilibrium we have $k_N^t = k_N$. This implies that there is a fixed supply of non-tradable goods denoted by $y_N = k_N + e_N$.

For the production of $T$ capital, there are competitive firms owned by the households, that transform tradable goods into $T$ capital in periods $t = 0, 1$. In order to produce $i_t \geq 0$ units of capital, the producers need $G(i_t)$ units of tradable goods. The function $G$ takes the form

$$G(i_t) = \phi_0 i_t + \frac{\phi_1}{1 + \eta} i_t^{1+\eta}.$$
The profits of the capital producing firms are

$$\Pi_t = \max_{i_t \geq 0} \Psi^T i_t - p^T_t G(i_t).$$  \hspace{1cm} (8)

Market clearing in the capital goods market in periods $t = 0, 1$ is given by

$$k^T_{t+1} = k^T_t + i_t,$$

as the capital inherited from past periods plus the newly produced capital is accumulated by banks for future production. In period $t = 2$ all capital fully depreciates after production has taken place and the capital goods market is not active.

### 2.1.4 Foreign investors

Foreign investors are risk neutral, consume only tradable goods, and discount the future with discount factor $\beta$. We assume that foreign investors can only buy claims denominated in foreign currency. Therefore, equilibrium in the domestic claims market requires $a_t = b_t$. On the other hand, in the foreign claims market the difference $a^*_t - b^*_t$ can be positive or negative, as foreign investors will absorb the difference.

Let $p^T_t$ denote the price of tradable goods in foreign currency, which is exogenous. The law of one price implies:

$$p^T_t = s_t p^T_t.$$

This price $p^T_t$ is normalized to 1 at $t = 0$ and is subject to random shocks at $t = 1, 2$. Specifically, at $t = 1$ the random variable $\varepsilon$ is realized and the price of non-tradables is permanently affected and equal to

$$p^T_1 = p^T_2 = 1/\varepsilon.$$

The variable $\varepsilon$ is lognormally distributed with mean 1 and variance $\sigma^2_\varepsilon$. This nominal disturbance will generate fluctuations in the nominal exchange rate in equilibrium, and it is introduced to get exchange rate movements that are orthogonal to the fundamentals of the domestic economy.

The price of foreign-denominated bonds is pinned down by the Euler equation of foreign investors

$$q^*_t = \beta E_t \left[ \frac{p^T_{t+2}}{p^T_{t+1}} \right] = \beta,$$

where
where the last equality follows from the stochastic properties of $p_{t}T^*$. 

### 2.1.5 Monetary regime and the nominal exchange rate

Our economy features flexible prices, so the only role of monetary policy is to determine nominal prices and the nominal exchange rate. The reason why these prices matter for the real allocation is that assets and liabilities are denominated in different currencies, so fluctuations in the nominal exchange rate reallocate wealth across agents.

For most of the analysis, we assume that the monetary authority is only concerned with price stability. Namely, we assume that the monetary authority successfully targets a constant CPI:

$$P_t = \bar{P} = \omega \cdot (1 - \omega)^{-(1 - \omega)}. \quad (10)$$

Combining this rule with the CPI definition (2) and the law of one price (9), we obtain the nominal exchange rate

$$s_t = \frac{1}{p_{t}^N} \times \left( \frac{p_{t}^T}{p_{t}^N} \right)^{1-\omega}. \quad (11)$$

Two forces drive the nominal exchange rate: nominal fluctuations in the price level in the rest of the world and movements in the relative price of tradables and non-tradables. Both forces will be relevant for our analysis.

### 2.2 Equilibrium

There are two sources of uncertainty in this economy, both realized at date $t = 1$. The nominal shock $\varepsilon$ introduced above, and a sunspot variable $\eta$ uniformly distributed in $[0,1]$. The sunspot will determine which equilibrium is played at $t = 1$ when multiple equilibria are possible. Note that we are leaving implicit in our notation that all variables dated 1 and 2 are function of the state of the world $(\eta, \varepsilon)$.

A competitive equilibrium is a vector of capital prices $\{\Psi_t^T, \Psi_t^N\}$, bond prices $\{q_t, q_t^*\}$, factor prices $\{R_t^T, R_t^N, w_t\}$, good prices $\{p_t^T, p_t^N, p_t^T^*\}$, nominal exchange rates $\{s_t\}$, asset and consumption choices for households $\{a_{t+1}, a_{t+1}^*, c_t^N, c_t^T\}$, portfolio choices for bankers $\{k_t^T, k_t^N, b_{t+1}, b_{t+1}^*\}$, and investment choices for the capital good producers $\{i_t, \hat{k}_t\}$, such that: (i) the choices of households, banks, and capital good producers solve their respective decision problem; (ii) all domestic markets clear; (iii) $q_t^*$ and $p_t^T^*$ are determined abroad; (iv) the law of one price holds; (v) and the price level $P_t$ is constant.
2.3 Discussion of assumptions

Let us briefly discuss the main simplifying assumptions made in the model.

First, we are making simplifying assumptions on the non-tradable sector: the non-tradable sector does not employ labor, the $N$ capital is in fixed supply, and the $N$ capital cannot be used as collateral. The first assumption simplifies the analysis as we don’t have determine how labor is allocated among the two sectors and the real wage will be immediately derived from the level of capital invested in the $T$ sector. The other two assumptions are convenient since they allow us to characterize all remaining equilibrium prices and quantities without solving for the price of the $N$ capital $\Psi^N_t$.

Second, we are assuming that foreign investors cannot trade domestic-currency claims. We could have a less stark form of segmentation, by allowing foreign investors to accept domestic-currency claims subject to some friction, as long as we don’t have an infinitely elastic demand for domestic claims. Ruling out foreign investors’ participation altogether is just a useful simplification.

Third, we are representing monetary policy purely as a choice of numeraire and we are assuming the monetary authority can commit to perfect price stability. This is a simple way to model a floating exchange rate regime, where nominal exchange rate volatility is not driven by inflationary choices of the central bank. As we shall see, our main mechanism is based on the relation between the country’s real wealth and the real exchange rate, so it is useful to mute other, policy-driven channels of exchange rate instability. In Section X, we will explore a version of the model that captures a pegged exchange rate regime, to study how our mechanism plays out in that case.

3 Twin crises with flexible exchange rates

We now show that the model can generate multiple equilibria. Specifically, we will focus on two types of outcomes, that we will label the “good” and the “bad” equilibrium. The former is supposed to mimic the behavior of the small open economy in normal times, while the latter is characterized by a twin crisis- a financial crisis that occurs in conjunction with a depreciation of the domestic currency. The main ingredient that allows the model to generate these bad equilibria is the households’ ability to change the currency denomination of their savings.

In the good equilibrium, households are willing to hold assets denominated in domestic currency. Thus, the domestic banking sector can finance its operations mostly by
issuing domestic currency liabilities. Because of that, fluctuations in the exchange rate have limited impact on the net worth of the banking sector, making the latter less prone to twin crises.

The bad equilibrium, instead, originates from households’ desire to hold assets denominated in foreign currency. When this motive is sufficiently strong, banks are forced to finance themselves mostly by issuing liabilities in foreign currency. As the currency mismatch in the banks’ balance sheet opens up, the domestic banking sector becomes more exposed to confidence crises, which take the form of depressed asset values, depressed economic activity and a depreciation of the exchange rate. This situation is self-fulfilling because households have a precautionary motive to denominate their savings in foreign currency when the prospects of a confidence crisis loom. Foreign currency assets are, in fact, a good hedge during a crisis because they appreciate precisely when households’ wealth plummets.

We analyze the model in two steps, moving backwards in time. First, we look at the economy starting in period 1, taking as given the capital stocks and financial claims inherited from period 0, and analyze how the equilibrium is determined in the last two periods. We call this a “continuation equilibrium,” and we show that for a subset of initial conditions there can be multiple continuation equilibria. Next, we go back to period $t = 0$ and study the equilibrium determination of investment and financial claims in that period. We then show examples in which equilibrium choices ex ante can lead to equilibrium multiplicity in the following periods.

3.1 Continuation equilibria

Consider the economy starting at $t = 1$, taking as given the values of the state variables $k_1^T, a_1, b_1, a_1^*, b_1^*$. For now, we do not need to introduce explicitly the sunspot variable $\eta$ and we simply ask whether, for given initial conditions, multiple equilibria are possible.

Our objective is to look for continuation equilibria using a simple diagram that plots two equilibrium relations between the price of capital in terms of tradables, $\psi_T^1 \equiv \frac{\Psi_T^1}{p_1^T}$, and the investment in $T$ capital by banks $k_2^T$—two relations that can be interpreted as demand and supply curves. As a preliminary step, however, we derive a relation between the variables $(\psi_T^1, k_2^T)$ and the equilibrium exchange rate $s_1$. This relation will be used to
determine the value of the banks’ liabilities in foreign currency.

### 3.1.1 Equilibrium exchange rate

The following lemma derives useful properties of a continuation equilibrium from the household Euler equations and the market clearing condition for non-tradable goods at $t = 1, 2$,

$$(1 - \omega) \frac{P_t C_t}{P_t^N} = y^N. \quad (12)$$

**Lemma 1.** All continuation equilibria satisfy the following conditions:

i. consumption is constant over time, $C_1 = C_2$;

ii. the relative price of non-tradable goods in terms of tradables and the prices of tradable and non-tradable goods in terms of the CPI are all constant over time,

$$\frac{p_N^1}{p_T^1} = \frac{p_N^2}{p_T^2}, \quad \frac{p_T^1}{P_1} = \frac{p_T^2}{P_2}, \quad \frac{p_N^1}{P_1} = \frac{p_N^2}{P_2}.$$  

iii. the domestic real interest rate is

$$\frac{1}{q_1 P_2} = \frac{1}{\beta}.$$  

The logic of the lemma is simple. Tradable consumption is perfectly smoothed by trading with foreign investors. Non-tradable consumption is constant because the non-tradable endowment is constant. So the relative price of tradables and non-tradables must be constant. The result for tradable and non-tradable prices in CPI terms follows from the definition of the CPI. The result for the domestic real interest rate comes from the Euler equation for domestic bonds.

Substituting constant consumption, constant relative prices, and the real bond prices $q_1^* = q_1 P_2 / P_1 = \beta$ in the household intertemporal budget constraint, we get:

$$C_1 = \frac{1}{1 + \beta} \left( \frac{w_1}{P_1} + \beta \frac{w_2}{P_2} + (1 + \beta) \frac{P_1^N}{P_1} e^N + \frac{\Pi_1}{P_1} + \frac{a_1 + s_1 a_1^*}{P_1} \right). \quad (13)$$

Consumption is proportional to household total wealth, which is equal to the present value of labor income and endowments, plus the profits of the capital producers, plus financial assets. To proceed, we substitute for real wages and for the profits of the capital producing firms, expressing both in terms of tradables. Real wages in tradables are equal
to the marginal product of labor,
\[
\frac{\bar{w}_t}{p_t} = (1 - \alpha)(k_t^T)^\alpha.
\]
Real profits in tradables can be rewritten defining the profit function \(\pi(\psi_t^T)\) as follows
\[
\frac{\Pi_t}{p_t^T} = \pi(\psi_t^T) = \frac{\eta}{1 + \eta \phi_t^N} \left(\psi_t^T - \phi_t^0\right)^{1+\eta\frac{\psi_t^T}{\phi_t^N}},
\]
which comes from solving (8). The market clearing condition for non-tradables (12) can then be rewritten, after rearranging, as
\[
1 - \omega \left\{ \frac{p_t^T}{p_t^N} \left[(1 - \alpha) \left(\frac{k_t^T}{k_t^2}\right)^\alpha + \beta(1 - \alpha) \left(\frac{k_t^T}{k_t^2}\right)^\alpha + \pi(\psi_t^T) + \epsilon a_1^* \right] + \left(\frac{p_t^T}{p_t^N}\right)^\omega a_1 \right\} + (1 - \omega) \epsilon^N = y^N, \tag{14}
\]
where the law of one price was used to substitute \(s_1 = \epsilon p_t^T\) and the monetary rule was used to express \(p_t^N\) in terms of \(p_t^N/p_t^T\).\(^4\)

Equation (14) identifies the first crucial mechanism in our model: more capital invested in the tradable sector \(k_t^T\) increases labor productivity and real wages; this shifts up the demand for non-tradables and leads to a real exchange rate appreciation (a higher value of \(p_t^N/p_t^T\)). This is just a version of the Balassa-Samuelson effect. But it plays an important role here because capital invested in the tradable sector depends on the health of the banks’ balance sheets—as we shall see shortly—and so this mechanism creates a positive feedback between banks’ balance sheets and real exchange rates.

From now on, we restrict attention to continuation equilibria with \(\alpha_1^* \geq 0\) to ensure that the non-tradable demand in (14) is everywhere decreasing and there is a unique value of \(p_t^N/p_t^T\) that solves (14). Given \(p_t^N/p_t^T\), we can fully determine household consumption, goods prices, and bond prices in the continuation equilibrium. These results are summarized in Lemma 2 below. The lemma also shows that \(p_t^N/p_t^T\) is increasing in \(\psi_t^T\) as long as \(\psi_t^T > \phi_t^0\).

**Lemma 2.** For any vector of initial conditions \((k_t^T, a_1, b_1, a_1^*, b_1^*)\) with \(a_1^* \geq 0\) and any values of the equilibrium variables \((\psi_t^T, k_t^2)\), there exists a unique vector of prices and quantities

\(^4\)A constant CPI requires \((p_t^T)^\omega (p_t^N)^{1-\omega} = 1\) which yields \(p_t^N = (p_t^N/p_t^T)^\omega\).
consistent with a continuation equilibrium. Let
\[
p_{1N} \over p_{1T} = h(\psi_{1T}, k^2_{T})
\] (15)
be the relation between \((\psi_{1T}, k^2_{T})\) and the relative price of non-tradables \(p_{1N} / p_{1T}\) from the mapping above. The function \(h\) is increasing in \(k^2_{T}\) and is increasing in \(\psi_{1T}\) if \(\psi_{1T} > \phi_0\).

We are now ready to analyze the determinants of \(\psi_{1T}\) and \(k^2_{T}\) and complete our characterization of a continuation equilibrium.

3.1.2 Supply and demand of capital goods

The supply of capital goods at \(t = 1\) is given by the optimization problem of capital producing firms (8). Rearranging the first order condition of that problem gives
\[
i_1 = \frac{1}{\phi_1} \left( \psi_{1T} - \phi_0 \right)^{1/\eta},
\] (16)
for \(\psi_{1T} \geq \phi_0\). If \(\psi_{1T} < \phi_0\) capital producers are at the corner \(i_1 = 0\). However, we will introduce assumptions that allow us to focus on the case \(\psi_{1T} \geq \phi_0\).

The demand for capital goods is determined by banks’ optimality. The rate of return to tradable capital invested at \(t = 1\) is \(R_{2T} / \Psi_{1T}\) because capital costs \(\Psi_{1T}\), earns the dividend \(R_{2T}\) at \(t = 2\), and then fully depreciates. The risk-free rate is \(1/q_1\). Two cases are then possible:

1. **Unconstrained banks.** The marginal gain from borrowing an extra unit of domestic currency and investing it in \(T\) capital is zero and the collateral constraint is slack,
\[
R_{2T} / \Psi_{1T} = \frac{1}{q_1}, \quad (1 - \theta) \Psi_{1T} k^2_{T} \leq [(\Psi_{1T} + R_{1T}) k_{1T} + p_{1N} k_{N} - b_1 - s_1 b^*_1].
\]

2. **Constrained banks.** The marginal gain from borrowing an extra unit of domestic currency and investing it in \(T\) capital is positive and the collateral constraint is binding,
\[
R_{2T} / \Psi_{1T} > \frac{1}{q_1}, \quad (1 - \theta) \Psi_{1T} k^2_{T} = [(\Psi_{1T} + R_{1T}) k_{1T} + p_{1N} k_{N} - b_1 - s_1 b^*_1].
\]

In the conditions above, we used equations (4)-(6) and equilibrium in the \(N\) capital market \(k^N_{t} = k^N\) to write the collateral constraint compactly.

In the unconstrained case, we can substitute for the rental rate from (3) and for \(q_1 = \beta\),
to obtain the unconstrained demand for capital:

\[ k^T_2 = \left( \frac{\alpha}{\psi^T_1} \right)^{\frac{1}{1-\alpha}}. \] (17)

In the constrained case, we can rewrite the binding collateral constraint in terms of tradables and obtain the constrained demand for capital:

\[ k^T_2 = \frac{1}{(1-\theta)\psi^T_1} \left[ \psi^T_1 k^T_1 + \alpha \left( k^T_1 \right)^a + \frac{p^N_1}{p^T_1} k^N - \left( \frac{p^N_1}{p^T_1} \right)^{1-\omega} b_1 - \varepsilon b^*_1 \right], \] (18)

where we used the monetary rule to obtain \( p^T_1 = \left( \frac{p^T_1}{p^N_1} \right)^{1-\omega} \). The presence of the relative price \( \frac{p^N_1}{p^T_1} \) in the constrained demand for capital captures the second crucial effect in our model: an appreciation of the real exchange rate (higher \( \frac{p^N_1}{p^T_1} \)) improves the balance sheet of the bank if most bank debt is denominated in foreign currency, i.e., if \( b_1 \) is low enough. The formal condition for a real appreciation to have a positive effect on the constrained demand for capital is:

\[ \frac{p^N_1}{p^T_1} k^N - (1-\omega) \left( \frac{p^N_1}{p^T_1} \right)^{1-\omega} b_1 > 0, \] (19)

and we will see its role shortly.

To analyze the problem in terms of the single price \( \psi^T_1 \) we define

\[ H(\psi^T_1) \equiv h \left( \psi^T_1, k^T_1 + \frac{1}{\varphi_1} \left( \psi^T_1 - \phi_0 \right)^{1/\eta} \right), \]

for all \( \psi^T_1 \geq \phi_0 \). Here we are using the supply of capital (16) and the market clearing condition \( k^T_2 = k^T_1 + i_1 \) to obtain \( p^N_1 / p^T_1 \) as a function of \( \psi^T_1 \) alone.

Substituting \( \frac{p^N_1}{p^T_1} = H(\psi^T_1) \) in (18) we obtain the constrained demand for capital for each price \( \psi^T_1 \). Taking the lowest value between the constrained and the unconstrained demand at each price \( \psi^T_1 \) gives us the demand curve for capital. Figure 1 shows this construction for a numerical example. We will discuss the example in more detail shortly.

First, let us derive an existence result. Define the real exchange rate at the price \( \psi^T_1 = \phi_0 \):

\[ h \equiv H(\phi_0). \]
Proposition 1. Assume the following inequalities are satisfied:

\[ \alpha \left( k^T_1 \right)^{\alpha - 1} > \phi_0, \quad \alpha \left( k^T_1 \right)^{\alpha} + h k^N + \theta \phi_0 k^T_1 > \frac{1}{2} \omega_1 b_1 + \varepsilon b^*_1. \] (A1)

Then there exists a continuation equilibrium with \( \psi^T_1 > \phi_0 \) for all realizations of \( \epsilon \).

From now on, we will focus on economies that satisfy (A1) and restrict attention to continuation equilibria with \( \psi^T_1 > \phi_0 \). The main advantage of these restrictions is that we do not need to worry about the possibility that banks have negative net worth and so we don’t need to specify how banks’ bankruptcy is resolved for bond holders. Of course, individual banks’ bankruptcies are commonplace in financial crises, but since here we are capturing the entire financial system in a single representative bank, it is easier to model a crisis as a severe reduction in the total net worth of the financial sector.

3.1.3 Equilibrium in the capital goods market

We can now combine the supply and demand relations just derived to study the equilibrium in the capital market. In Figure 2 we plot demand and supply for a numerical example. As explained earlier, the demand for capital goods is non-monotone. In the un-
Figure 2: Equilibrium in the capital market

constrained region, the lower the price of capital goods the higher the demand of capital by the bankers. In the constrained region, instead, demand increases with the price. The supply of capital is, instead, upward sloping, with the flat region arising because capital good producers have no incentives to supply capital goods when the price falls below $A$.

Importantly, the relation between the demand for capital goods and their price changes when the collateral constraint binds. When $\Psi_T$ declines, the banks experience balance sheet losses, and the associated decline in their net worth reduces the resources that banks have to buy capital goods. This balance sheet channel is amplified by the equilibrium behavior of the exchange rate. As $k_2^T$ declines, the domestic currency depreciates and banks’ net worth declines further if the banks entered the period with liabilities in foreign currency, $b_1^* > 0$. Thus, a decline in the price of capital could be associated to a decline in the demand for capital goods when the collateral constraint binds in period 1. As we will discuss momentarily, this non-monotonicity in the demand schedule for capital goods can lead to multiple continuation equilibria.

In the left panel of Figure 1 we describe a situation where there is only one pair of prices and quantities consistent with market clearing. Given this pair, we can derive uniquely all other quantities and prices at $t = 1$ and $t = 2$. Therefore, in this case, the model features
only one continuation equilibrium.

In the right panel of the figure, instead, we describe a situation in which more than one equilibrium can arise in the capital market. We can see that there are three points in which the demand of capital crosses the supply schedule. Point A describes an equilibrium in which banks are unconstrained, while in point B and C the price of capital is so low that the collateral constraint of the banks binds. Associated to these equilibria in the capital market, there are different levels of aggregate consumption and of the exchange rate. As discussed earlier, households’ consumption increases in $k_{2T}$, while domestic currency appreciates the higher the capital stock in the tradable sector. Therefore, the equilibria are Pareto-ranked, in the sense that households’ and banks’ utility are the highest in the unconstrained equilibrium, and they decline monotonically with the prices and quantities that clear the capital market. Note also that some of the equilibria may be “unstable”, for example the one in point B in the figure.

The model can thus feature multiple continuation equilibria, and which one will be played depends on agents’ expectations. When agents expect asset prices to be high, banks are unconstrained, and their demand of capital is high. This translates into high consumers’ wealth and high aggregate consumption. When agents expect low asset prices, instead, the economy will experience a crisis: banks’ financial wealth plummets, their demand for productive assets declines, leading to a decline in the wealth of households and in their consumption. We select among these possible outcomes using a sunspot. In going forward, we will assume that agents never coordinate on an unstable continuation equilibrium, and that the economy features at most two stable continuation equilibria. When confidence crisis are possible, we will assume that with probability $\pi$ the agents will play the “bad” (constrained) equilibrium, and with probability $(1 - \pi)$ they will play the “good” (unconstrained) one.

The existence of these multiple continuation equilibria depends on models’ parameters and on the initial asset positions that bank’ and households inherit in period 1. Before closing this section it is important to briefly comment on the role played by the banks’ initial balance sheet. As we have discussed earlier, exchange rates depreciates when the capital stock declines, and this amplifies the negative effects that a drop in asset prices has on the demand for capital when the banks are financially constrained. From equation (??), we can see that this amplification is stronger the higher $b_1^{*}$, the more foreign currency debt banks owe. We can illustrate this point graphically by considering the equilibrium in the capital market for two sets of initial conditions. In the first case, banks enter period 1 with only domestic currency liabilities ($b_1 > 0, b_1^{*} = 0$), while in the second case they inherit only foreign currency liabilities ($b_1 = 0, b_1^{*} > 0$). We adjust the level of $b_1$ and $b_1^{*}$ so that
banks have the same leverage in these two cases, and we keep all remaining parameters and initial conditions fixed.

**Figure 3: The role of the banks’ balance sheet**

In Figure 3 we plot the demand schedules in these two scenarios. The presence of foreign currency liabilities for the banking sector flattens the upward sloping portion of the demand schedule. When banks do not inherit foreign currency liabilities, movements in the exchange rate have little impact on the banks’ net worth. When banks have debt denominated in foreign currency, instead, the depreciation of the exchange rate that is set in motion by the decline in the price of capital have more dramatic effects on banks’ net worth and on the demand of productive capital in the economy. This channel, that distinguish our environment from that of a closed economy, can be sufficiently strong to expose the economy to the confidence crises studied in this section. Indeed, in the figure presented above, the twin crises equilibria arise only when banks enter the period with foreign currency liabilities, and not otherwise.

This latter discussion makes clear that the portfolio choices of households and banks in period 0 determine whether the economy is exposed to confidence crises in the continuation equilibria. We now turn to the analysis of such choices.

Equation (??) implicitly defines a relation between $s$ and $k^t_1$. When banks invest more in productive capital, households’ wages in period 2 increase. Therefore, households’ consumption increase. The associated increase in the demand for non-tradable goods leads to
an increase in their price by equation (??). Because the monetary authority is committed
to price stability, the price of domestic tradable goods declines, and the exchange rate
appreciates. The opposite pattern emerges when capital in the tradable sector declines.
Therefore, in the continuation equilibrium, the exchange rate depreciates when banks’ in-
vestment declines. We summarize this equilibrium relationship by writing the exchange
rate as a function of the capital stock in the tradable sector, \( s(k_T^T) \).

3.2 Optimal portfolio choices at \( t = 0 \)

We start analyzing the portfolio choices of households at \( t = 0 \). The Euler equations
characterizing their behavior are given by

\[
q_0 U'(C_0) = \mathbf{E}_0[U'(C_1)], \tag{20}
\]
\[
s_0 q_0^* U'(C_0) = \mathbf{E}_0[s_1 U'(C_1)]. \tag{21}
\]

Differently from the case analyzed in the previous section, households at \( t = 0 \) face risk.
This risk takes two forms. First, the price of foreign tradable goods is stochastic, and these
shocks to \( p_T^T \) generate fluctuations in the exchange rate. Second, there might be the risk
of a twin crisis in the future. Given our selection rule, the economy experiences such a
crisis with probability \( \pi \) if the balance sheet inherited by the banks at \( t = 1 \) exposes the
economy to multiple continuation equilibria.

These two sources of risk have different implications for the households’ choices re-
garding the currency composition of their assets. To illustrate this point, we can combine
equations (20) and (21) as follows,

\[
\mathbf{E}_0[R_{fc}^1] - R_{dc}^0 = -\mathbf{Cov}_0 \left[ R_{fc}^1, U'(C_1) \right], \tag{22}
\]

where \( R_{fc}^1 = s_1/(s_0 q_0^*) \) are the \( t = 1 \) realized returns for bonds denominated in foreign
currency, and \( R_{dc}^0 = 1/q_0 \) are the returns for bonds denominated in domestic currency. Equation (22) is a standard asset pricing condition that determines the yields on foreign
currency assets relative to those on assets denominated in domestic currency.

Consider first a scenario in which only the unconstrained continuation equilibrium is
played from \( t = 1 \) onward.\(^5\) In such a case, shocks to foreign tradable goods affect the
exchange rate, but do not impact the equilibrium level of the capital stock, and they are

\(^5\)This happens if the asset structure at \( t = 1 \) does not permit multiple continuation equilibria, or if \( \pi = 0 \).
thus orthogonal to households’ labor income and to the profits of capital good producers. Because of that, households have an incentive to save in domestic currency because foreign currency bonds increase the volatility of their consumption at $t = 1$. This can be formally seen by inspecting $t = 1$ consumption in the unconstrained continuation equilibrium, see equation (22). When $a_1^* > 0$, $C_1$ will be positively associated to the equilibrium exchange rate $s_1$. Therefore, $\text{Cov}_0 \left[ R_{fc}^1, \frac{U'(C_1)}{U'(C_0)} \right] < 0$ when $a_1^* > 0$, and by equation (22) we must have that foreign currency bonds pay a risk premium to make households willing to hold them.

The prospect of a crisis at $t = 1$, however, generates an incentive for households to denominate their savings in foreign currency. To illustrate this point, consider a second scenario in which the twin crisis equilibrium is played at $t = 1$ with probability $\pi > 0$, and assume for simplicity that $\sigma_\epsilon = 0$. In this case, there are only two states of the world at $t = 1$. With probability $(1 - \pi)$, the economy is in the good continuation equilibrium, with high consumption and a relatively appreciated exchange rate. With probability $\pi$, the economy experiences a twin crisis: households’ consumption is low and the exchange rate relatively appreciated. Foreign currency bonds are a good crisis hedge from the households’ perspective because they pay a high return in the bad state of the world, which implies $\text{Cov}_0 \left[ R_{fc}^1, \frac{U'(C_1)}{U'(C_0)} \right] > 0$. In equilibrium, households are willing to accept lower yields on foreign currency bonds because of their hedging property.

While households wish to save in foreign currency when they expect a twin crisis in the future, banks’ have the opposite incentive: they would like to issue debt denominated in domestic currency. To understand this point in the simplest possible way, we assume that initial conditions at $t = 0$ are such that the banks’ collateral constraint does not bind, and we assume that the banks cannot invest/disinvest in capital, $k_T^0 = k_T^1$. Hence, their problem consists in choosing the currency composition of their liabilities, $b_1$ and $b_1^*$. The first order conditions for the banks at $t = 0$ are similar to those of the households,

\begin{align*}
q_0 \lambda_0 &= E_0[\lambda_1], \\
s_0 q_0^* \lambda_0 &= E_0[s_1 \lambda_1],
\end{align*}

where $\lambda_t$ is the banks’ marginal value of wealth at time $t$. The marginal utility of wealth for the banks may be stochastic at $t = 1$ even though the banks are risk neutral and consume only in the final period. This is due to the possibility that the collateral constraint may bind in period 1.\textsuperscript{6} When the collateral constraint does not bind at $t = 1$, the value of one

\textsuperscript{6}See, for example, Aiyagari and Gertler (1999) and Bocola (2016).
unit of domestic currency at \( t = 1 \) equals

\[
\lambda_1^{\text{not bind}} = \frac{1}{q_1 p_2^T}.
\]  

(25)

That is, the banks can use that unit of net worth to invest, obtain the return \( 1/q_1 \) at \( t = 2 \), and convert the proceeds into tradable goods. When the collateral constraint binds at \( t = 1 \), instead, the value of one unit of domestic currency at \( t = 1 \) equals

\[
\lambda_1^{\text{bind}} = \frac{1}{(1 - \theta) \Psi_1^T} \left[ \frac{R_2^T - \theta \Psi_1^T / q_1}{p_2^T} \right].
\]  

(26)

The banks can in fact lever that unit of net worth up to \( 1/(1 - \theta) \Psi_1^T \), invest in tradable capital, and obtain a return of \( \left[ R_2^T - \theta \Psi_1^T / q_1 \right] \) tradable goods in period \( t = 2 \).

Importantly, one can verify that \( \lambda_1^{\text{bind}} \geq \lambda_1^{\text{not bind}} \): a unit of net worth allows the banks to relax their collateral constraint, and it is more valuable when the latter binds. Rearranging equations (23) and (24), we then have

\[
\mathbb{E}_0 [R_1^{fc} - R_0^{dc}] / R_0^{dc} = -\text{Cov}_0 \left[ R_1^{fc}, \lambda_1 \right] / \lambda_0.
\]  

(27)

This expression clarifies that banks have little incentives to issue foreign currency debt when they anticipate a twin crisis at \( t = 1 \). A twin crisis is, in fact, associated to binding collateral constraints for the banks, implying an high \( \lambda_1 \). Moreover, it is a state of the world in which the domestic currency depreciates, and this raises the payments that banks have to make on their foreign currency debt. Therefore, banks have a precautionary motive to issue domestic currency debt when a twin crisis is anticipated.

### 3.3 Self-fulfilling twin crises

Having discussed optimal portfolio choices at \( t = 0 \), we can now turn to the analysis of the competitive equilibria in the model. Specifically, we ask whether the portfolio choices of banks and households at \( t = 0 \) can expose the economy to the bad continuation equilibria at \( t = 1 \), or whether they prevent these crises from happening. As we argue below, the answer to this question depends on the risk aversion of the households relative to the (implicit) risk aversion of the banks.

In order to understand why, suppose that at \( t = 0 \) agents attach a high likelihood to a future twin crisis. As explained earlier, households have an incentive to save in foreign
currency while banks have a precautionary motive to issue debt in domestic currency. If the former effect dominates, the banks will have to issue foreign currency liabilities at $t = 0$. The presence of foreign currency liabilities in the banks’ balance sheet could then expose the financial sector to confidence crises at $t = 1$. This would validate agents’ expectation of a twin crisis at $t = 1$. On the contrary, when banks’ precautionary motives are sufficiently strong, the expectation of future twin crises may lead to a reduction of their foreign currency liabilities. This would make the economy less exposed to the twin crises continuation equilibria studied in Section 3.1.

In what follows, we construct a numerical example and show that the model is indeed capable of generating self-fulfilling twin crises. Specifically, we adopt the following utility function for households,

$$U(C_t) = \frac{(C_t - \bar{C})^{1-\sigma}}{1-\sigma}.$$  

Households’ risk aversion could therefore be extremely high when consumption gravitates around the consumption commitment $\bar{C}$.

We can now turn to the problem of deriving a competitive equilibrium. We have already determined all quantities and prices from $t = 1$ onward. Therefore, we just need to determine the $t = 0$ portfolio choices $\{a_1, a_1^*, b_1, b_1^*\}$, the prices $\{p_N^0, p_T^0, q_0\}$, the exchange rate $s_0$, and households’ consumption $C_0$. The equilibrium conditions these variables need to satisfy are the Euler equations of households and banks (20)-(24), their budget constraints, market clearing in the non-tradable good sector, the law of one price, and market clearing in domestic currency bond market,

$$a_1 = b_1.$$  

We can use all these equilibrium conditions and collapse the problem into that of choosing a $b_1^*$ such that the Euler equations of banks and households are both satisfied,

$$\mathbb{E}_0[s_1 U'(C_1)] \mathbb{E}_0[s_1 \lambda_1] = \mathbb{E}_0[s_1] \mathbb{E}_0[s_1^2 \lambda_1].$$  

In Figure 4 we plot the left and right hand side of equation (29) as a function of the foreign currency liabilities that the banking sector takes in period 0. As we move $b_1^*$, the asset structure in the economy adjusts in order to fulfill the remaining equilibrium conditions. Specifically, by the bankers’ budget constraint, high $b_1^*$ is associated to a low debt in domestic currency and, by equation (28), to a low level of households’ savings in domestic currency. We can verify that the two curves cross twice, demonstrating the
existence of multiple continuation equilibria.

Figure 4: **Portfolio choices and multiple equilibria**

![Graph showing portfolio choices and multiple equilibria](image)

Let’s consider first the equilibrium at the low level of $b_1^*$. In our example, the banking sector is not exposed to confidence crisis at $t = 1$ when $b_1^*$ is low. Hence, from period $t = 1$ onward, the economy operates only in the good continuation equilibrium, and the only source of risk for this economy arises because of the fluctuations in the price of foreign tradable goods. Because of that, we know from our previous discussion that households are not willing to save in foreign currency unless foreign currency debt provides a risk premium. However, neither foreigners nor banks wishes to pay such premium, as they both act as risk neutral in this equilibrium. Therefore, households save exclusively in domestic currency, $a_1 > 0$ and $a_1^* = 0$.

These choices by the households allows the banks to issue in the first place mostly domestic currency assets, sustaining this outcome as an equilibrium of the model.

In the second equilibrium, instead, the banks issue large amounts of foreign currency liabilities. In our example, the large amount of foreign currency debt exposes the banking sector to a confidence crisis at $t = 1$. Therefore, households now face the risk of a twin crisis at $t = 1$. We know from the previous discussion that foreign currency assets have good hedging properties during a crisis, and this pushes households to save mostly in

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7 We can verify this in Figure 1, as $E_0[s_1U'(C_1)] = E_0[s_1]E_0[U'(C_1)]$. This means that $s_1$ and $U'(C_1)$ are orthogonal in the continuation equilibria, a situation that occurs only when $a_1^* = 0$. 

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foreign currency. The scarcity of savings in domestic currency forces the banking sector to finance its operation by issuing foreign currency liabilities. As the “dollarization” of banks’ liabilities occurs, the economy becomes exposed to the twin crisis equilibria in period 1.

This second equilibrium captures, in our view, salient features of the financial fragility for economies with an open capital account and flexible exchange rates. The economy may operate most of the time in the good equilibrium, where the domestic financial sector does not have hard time financing itself in domestic currency. However, the expectation of a confidence crisis may suddenly lead households to “run” on the domestic currency and redenominate their assets in foreign currency. This capital flights increase the fragility of the banking sector and they contribute to further expose the economy to a confidence crisis. We now turn to analyze how the interactions of capital flights and financial crisis affects the operations of lending of last resort for an open economy.

4 Lending of Last Resort

We consider a government that at \( t = 0 \) levies a lump sum tax \( \tau_0 \) on the households. The government can use these resources to accumulate reserves in foreign and in domestic currency,

\[ q_0 h_1 + s_0 q_0^* h_1^* = \tau_0. \]

To make the problem interesting, we assume that the government faces some upper bound in the taxes that it can collect from households, \( \tau_0 \leq \bar{\tau} \).

The reserves can be used in period 1 to conduct operations of lending of last resort. Specifically, we assume that the government can commit to a demand schedule for capital goods at \( t = 1 \).\(^8\) We denote by \( z_1(\Psi^T) \) the demand of capital goods by the government given the price \( \Psi^T \),

\[ \Psi^T z_1(\Psi^T) \leq h_1 + s_1 h_1^*. \]

The government holds the capital stock for one period and operates an alternative linear technology that returns \( A z_1 \) units of tradable goods in period 2. These resources are then rebated back to the consumers as a transfer in period 2.

These operations can potentially eliminate the bad continuation equilibria at \( t = 1 \). By adding to private capital demand, the government can curb the fall in asset prices and the

\(^8\)Equivalently, we could have modeled operation of lending of last resort as a direct loan that the government extends to the bank at some penalty rate.
depreciation of the exchange rate that characterizes the bad equilibrium, limiting in this fashion the decline in the banks’ net worth that sustains the bad equilibria. However, to be effective, they must also be credible. That is, the government needs to have enough fiscal resources to shift out the demand for capital and eliminate the bad equilibria.

In order to formalize this last point, we can define $Z(\Psi_T)$ to be the difference between the supply of capital and its private demand when the price is $\Psi_T$. The government can credibly eliminate the bad continuation equilibria if

$$\Psi_T Z(\Psi_T) \leq h_1 + s_1 h_1^*,$$

for all $\Psi_T$ that are below the good equilibrium price.

Figure 5 clarifies this discussion. In the left panel we can see that $Z(\Psi_T)$ is the excess supply of capital in the bad equilibrium of the model. If the government has enough reserves to purchase $Z(\Psi_T)$ at the market prices $\Psi_T$, it could prevent the confidence crisis in period 1: by committing to purchase capital goods, the government can shift the demand schedule to the right and implement uniquely the good equilibrium, see the right panel in Figure 5 for an example of a successful operation. If the government does not have enough fiscal resources, however, the bad equilibrium is unavoidable and lending of last resort is less effective.

Figure 5: Effective lending of last resort

While this section of the paper is still largely a work in progress, we wish to point out
two results that naturally emerge in our environment. First, the possibility of the capital flights makes it harder for the government to credibly avert a confidence crisis in period 1 through lending of last resort. As we have seen earlier, the presence of foreign currency liabilities for banks flattens the capital demand schedule in its upward sloping portion, see Figure 3. *Ceteris paribus*, this implies an increase in the amount of fiscal resources necessary to avert the bad equilibrium, $\Psi_T Z(\Psi_T^T)$. This is the sense in which an open economy with flexible exchange rates makes effective lending of last resort more difficult.

Second, the model provides a rationale for the ex-ante accumulation of foreign reserves. Foreign reserves have the property of appreciating in the bad equilibrium. As such, they increase the fiscal resources that the government has at its disposal to avert the bad equilibrium, and to effectively conduct lending of last resort. This last result can rationalize the findings of Obstfeld, Shambaugh, and Taylor (2010) and Aizenman and Lee (2007) that the large accumulation of foreign reserves among emerging markets over the last 20 years is strongly associated to the extent of financial openness and financial depth of these economies, and the informal arguments brought in the literature that such accumulation was done in order to facilitate effective lending of last resort.

5 Conclusion

[to be completed]
References


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6 Appendix

6.1 Proof of Lemma 1

Since there is no uncertainty left after period 1 the Euler equations for domestic and foreign bonds are

\[ q_1 U'(C_1) = \frac{P_1}{P_2} \beta U'(C_2), \] (30)

\[ q_1^* U'(C_1) = \frac{s_1 P_1}{s_2 P_2} \beta U'(C_2). \] (31)

Using the market clearing condition in the non-tradable market (12), the definition of CPI (2), and the constant endowment of non-tradables, we get

\[ (1 - \omega) \left( \frac{p_T}{p_N} \right) \omega C_1 = (1 - \omega) \left( \frac{p_T}{p_N} \right) \omega C_2. \]

Using \( q^* = \beta \), the law of one price (9), the assumption of constant foreign prices, and the definition of CPI (2), the Euler equation for foreign bonds (31) can be rewritten as

\[ U'(C_1) = \frac{\left( \frac{p^N_1}{p^T_1} \right)^{1-\omega} \left( \frac{p^N_2}{p^T_2} \right)^{1-\omega} U'(C_2)}. \]

Combining these two conditions yields

\[ (U'(C_1))^{-\frac{\omega}{1-\omega}} C_1 = (U'(C_2))^{-\frac{\omega}{1-\omega}} C_2, \]

which, given the concavity of \( U \), implies \( C_1 = C_2 \). The preceding equation implies a constant relative price of non-tradables. The goods prices in terms of CPI are monotone transformations of \( \frac{p^N_1}{p^T_1} \), so they are also constant. The domestic bond price comes from (30).
6.2 Proof of Lemma 2

All the steps are in the text except for the comparative statics with respect to $\psi_1^T$, which is straightforward.